Name: _____

AP Calculus I/AB and Calculus – Summer 2016
Summer Assignment Review Packet – ARE YOU READY FOR CALCULUS?

Calculus is a VERY RIGOROUS course
and completing this packet with your best effort
will help you to succeed.

This packet is to be completed and is due on the first day you return to school in September. All work for Part I is to be done neatly and orderly on your own paper while Parts II and III are to be completed in this packet .You will be tested on this material in your AP Calculus I/AB or Calculus class in September.

This review packet is meant to help you review the algebra, geometry, and pre-calculus (math analysis) concepts you will need in order to have success in your Calculus experience.

This packet contains questions assessing topics THAT YOU HAVE ALREADY BEEN TAUGHT. These topics will NOT be re-taught during your Calculus course. If there are any topics in this packet that you do not remember how to do you should work on them over the summer using any resources available (working groups, asking for help from family and/or friends, tutoring, internet sites, review workbooks, independent study, …)

Do not wait until the end of the summer to look at this packet. You should work on this packet a little at a time. This will allow you to take the necessary time to review topics that you are having trouble remembering how to do. You are responsible to review these concepts if you have not mastered them yet.

Here are some websites that may be of some help:

Calculus Textbook Appendices
http://college.cengage.com/mathematics/larson/calculus_analytic/7e/students/appendices.html

Precalculus Lessons at:
Youtube - search topics under brightstorm.

Quiz Questions and Review at:
http://www.math.buffalo.edu/rur/rurci3.cgi

And of course there is always
Ask Dr. Math at:
http://mathforum.org/dr.math/

Khanacademy.com

Note: Except for # 32, 34 and 36 calculator use is NOT permitted to solve any of the problems.

Part I

1. Simplify: (a) $\frac{x^3-9x}{x^2-8x+15}$ (b) $\frac{x^2-3x-10}{x^3+x^2-2x}$ (c) $\frac{\frac{1}{x}-\frac{1}{4}}{\frac{1}{x^2}-\frac{1}{16}}$ (d) $\frac{9-x^{-2}}{3+x^{-1}}$

2. Rationalize the denominator: (a) $\frac{2}{\sqrt{5}+\sqrt{2}}$ (b) $\frac{4}{1-\sqrt{3}}$ (c) $\frac{-3}{\sqrt{2}-\sqrt{5}}$

3. Write each of the following expressions in the form $c\,a^p b^q$ where c, p and q are numbers:

   (a) $\frac{(2a^2)^3}{b}$ (b) $\sqrt{9ab^3}$ (c) $\frac{(a^{-2})}{(b^{-2})} * (\frac{b}{a})^3$ (d) $\frac{12(x+y)^3}{9(x+y)}$ (e) $\frac{a^{-2}}{(b^{-1})\sqrt{a}}$ (f) $\left(\frac{a^{2/3}}{b^{1/2}}\right)^2\left(\frac{b^{3/2}}{a^{1/2}}\right)$

   (g) $\frac{5^{\frac{-1}{2}}.5x^{\frac{5}{2}}}{(5x)^{\frac{3}{2}}}$ (h) $\frac{(2x^2)^{\frac{3}{2}}}{2^{\frac{1}{2}}x^4}$

4. Solve for x (do not use a calculator)

   (a) $3^{(x+1)} = 27$ (b) $\frac{1}{9} = 3^{2x+2}$ (c) $\log_2 x = 3$ (d) $\log_3 x^2 = 2\log_3 4 - 4\log_3 5$

5. Simplify: (a) $\log_2 5 + \log_2(x^2-1) - \log_2(x-1)$ (b) $2\log_4 9 - \log_2 3$ (c) $3^{2\log_3 5}$

6. Simplify: (a) $\log_{10}(10)^{\frac{1}{2}}$ (b) $\log_{10}(\frac{1}{10^x})$ (c) $2\log_{10}\sqrt{x} + 3\log_{10}(x)^{\frac{1}{3}}$

7. Solve the following equations for the indicated variables.

(a) $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$, for a (b) $V = 2\,(ab + bc + ca)$, for a

(c) $A=2\pi r^2+2\pi rh$, for r (Use Quadratic formula to solve for r) *(d)* $A = P + nrP$, for P

(e) $2x - 2yd = y + xd$, for d (f) $\frac{2x}{4\pi}+\frac{1-x}{2}=0$, for x

8. For the equations

(a) $y = x^2 + 4x + 3$ (b) $3x^2 + 3x + 2y = 0$ (c) $9x^2 - 6y - 9 - x = 0$

complete the square and reduce to one of the standard forms $y - b = A(x - a)^2$ or $x - a = A(y - b)^2$.

9. Factor completely: (a) $x^6 - 16x^4$ (b) $4x^3 - 8x^2 - 25x + 50$ (c) $8x^3 + 27$ (d) $x^4 - 1$

10. Find all real solutions to:

(a) $x^6 - 16x^4 = 0$ (b) $27x^3 - 64 = 0$

(c) $25x+8x^2-4x^3=50$ (d) $x^3-x^2-13x-3=0$

11. Solve for x:

(a) $3\sin^2 x=\cos^2 x;\ 0\le x<2\pi$ (b) $\cos^2 x-\sin^2 x=\sin x;\ -\pi<x\le\pi$

(c) $\tan x+\sec x=2\cos x;\ -\infty<x<\infty$

12. Without using a calculator, evaluate the following:

(a) $\cos 210°$ (b) $\sin\frac{5\pi}{4}$ (c) $\tan^{-1}(-1)$ (d) $\sin^{-1}(-1)$

(e) $\cos\frac{9\pi}{4}$ (f) $\sin^{-1}\frac{\sqrt{3}}{2}$ (g) $\tan\frac{7\pi}{6}$ (h) $\cos^{-1}(-1)$

13.) Find the product

(a) $(2x-8)^2$

(b) $(3x+4)^2$

(c) $(5x+3)(2x-1)$

(d) $(3x-2y)(7x+8y)$

14. Solve the equations: (a) $4x^2 + 12x + 3 = 0$ (b) $2x+1=\frac{5}{x+2}$ (c) $\frac{x+1}{x}-\frac{x}{x+1}=0$

15. Find the remainders on division of
(a) $x^5 - 4x^4 + x^3 - 7x + 1$ by $x + 2$ (b) $x^5 - x^4 + x^3 + 2x^2 - x + 4$ by $x^3 + 1$

16. (a) The equation $12x^3 - 23x^2 - 3x + 2 = 0$ has a solution $x = 2$. Find all other solutions.

(b) Solve for x, the equation $12x^3 + 8x^2 - x - 1 = 0$.

(All solutions are rational and between ±1.)

17. Solve the inequalities: (a) $x^2+2x-3\le 0$ (b) $\frac{2x-1}{3x-2}\le 1$ (c) $x^2+x+1>0$

18. Solve for x: (a) $|-x+4|\le 1$ (b) $|5x - 2| = 8$ (c) $|2x + 1| = x + 3$

19. Determine the equations of the following lines:

(a) the line through (-1,3) and (2,-4);
(b) the line through (-1,2) and parallel to the line $2x - 3y + 5 = 0$,
perpendicular to the line $2x - 3y + 5 = 0$;
(c) the line through (2,3) and the midpoint of the line segment from (-1,4) to (3,2).

20. (a) Find the point of intersection of the lines: $3x - y - 7 = 0$ and $x + 5y + 3 = 0$.

(b) Shade the region in the xy-plane that is described by the system of inequalities. $\begin{aligned} 3x - y - 7 &< 0 \\ x + 5y + 3 &\geq 0 \end{aligned}$

21. Find the equations of the following circles:

(a) the circle with center at (1,2) that passes through the point (-2,-1)

(b) the circle that passes through the origin and has intercepts equal to 1 and 2 on the x- and y- axes, respectively.

22. For the circle $x^2 + y^2 + 6x - 4y + 3 = 0$, find:
(a) the center and radius; (b) the equation of the tangent at (-2,5).

23. A curve is traced by a point $P(x,y)$ which moves such that its distance from the point

A(-1,1) is three times its distance from the point B(2,-1). Determine the equation of the curve.

24. (a) Find the domain of the function $f(x) = \dfrac{3x+1}{\sqrt{x^2 + x - 2}}$

(b) Find the domain and range of the functions: i) $f(x) = 7$ ii) $g(x) = \dfrac{5x-3}{2x+1}$

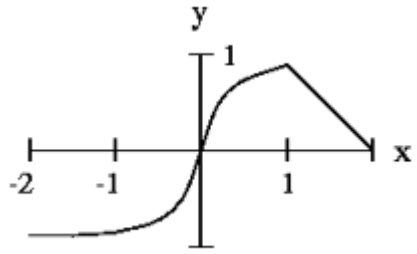
25. Let $f(x) = \dfrac{|x|}{x}$, show that $f(x) = \begin{cases} 1, & x > 0 \\ -1, & x < 0 \end{cases}$

Then, find the domain and range of $f(x)$.

26. Evaluate the value of for each functions at given points f(-1), f(3x), f(x+2) and f(x+h) , where

(a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{1}{x+1}$ (c) $f(x) = x^2$

27. The graph of the function $y = f(x) is\ given\ as\ follows$



Sketch the graphs of the functions

(a) $f(x+1)$ (b) $f(-x)$ (c) $|f(x)|$ (d) $f(|x|)$

28. Sketch the graphs of the functions: (a) $g(x) = |3x + 2|$ (b) $h(x) = |x(x-1)|$

29. (a) The graph of a quadratic function(a parabola) has x-intecepts -1 and 3 and a range consisting of all numbers less than or equal to 4. Determine an expression for the function.

(b) Sketch the graph of the quadratic function $y = 2x^2 - 4x + 3$

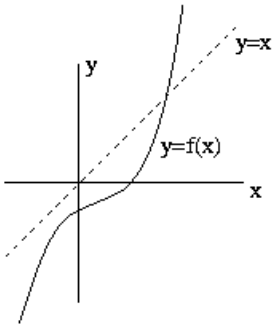
30. Find the inverse of the functions:

(a) $f(x) = 2x + 3$ (b) $f(x) = \dfrac{x+2}{5x-1}$ (c) $f(x) = x^2 + 2x - 1,\ x > 0$

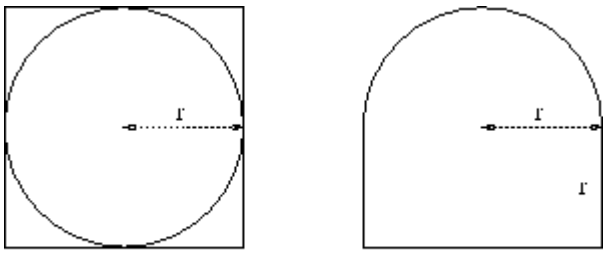
d) How do you know a function has an inverse?

31. A function f(x) has the graph to the right

Sketch the graph of the inverse function ($f^{-1}(x)$



32.)(a) Find the ratio of the area inside the square but outside the circle to the area of the square in the picture (a) below.



(a) (b)

(b) Find a formula for the perimeter of a window of the shape in the picture (b) above.

(c) Two cars start moving from the same point. One travels south at 100km/hour, the other west at 50 km/hour. How far apart are they two hours later?

(d) A kite is 100m above the ground. If there are 200m of string out, what is the angle between the string and the horizontal. (Assume that the string is perfectly straight.)

33. Using the trig identities:

(a) Show that $\frac{\sin\theta}{1+\cos\theta}+\frac{\cos\theta}{\sin\theta}$ can be simplified to $\csc\theta$

(b) Rewrite $\frac{1}{1+\sin x}$ so that it is not in fractional form.

(c) Show that $\frac{\sec^2\theta-1}{\sec^2}$ can be simplified to $\sin^2\theta$

34. Use Binomial Theorem Expansion (do NOT use repeated multiplication) to expand the following:

(a) $(x-2)^4$ (b) $(3x - 2y)^6$

35. Determine the right-hand and left-hand behavior of the graph of:

(a) $f(x)=1+2x-x^6$ (b) $f(x)=-2.1x^5+4x^3-2x^2+138.$

36. Two planes start from the same airport and fly in opposite directions. The second plane starts one-half hour after the first plane, but its speed is 80 kilometers per hour faster. Find the air speed of each plane if 2 hours after the first plane departs the planes are 3200 kilometers apart.

37. The path of a diver is given by $y = -\frac{4}{9}x^2 + \frac{24}{9}x + 12$ where *y* is the height in feet and *x* is the horizontal distance from the end of the diving board in feet. What is the maximum height of the dive?

38. (a) Rewrite using double angle $F(x) = \sin ax \cos ax$

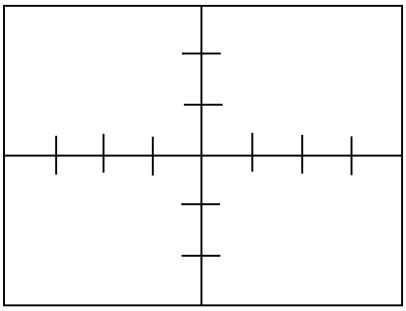
(b) What are the three Pythagorean Triples of trig?

Part II

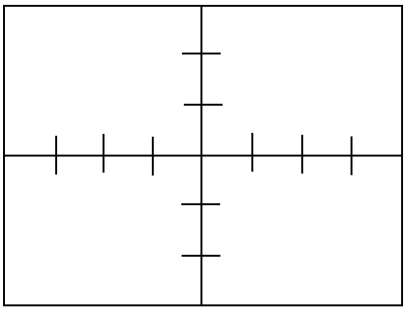
Graphical Analysis – Much of Calculus deals with functions and their graphical characteristics. To facilitate the study of functions, it is important that you be able to quickly sketch the graph of a basic function or a transformation of a basic function ***without*** a calculator.

Without using a calculator, graph each function below. As always, it is necessary to indicate the units and label the axes provided. Full credit will not be earned without units and axes completely labeled.

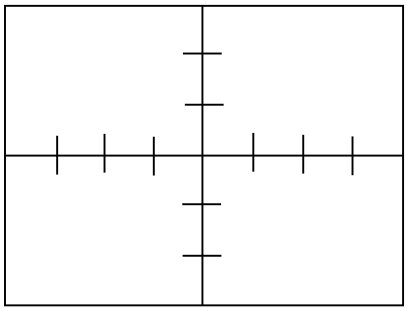
$f(x) = x^2$



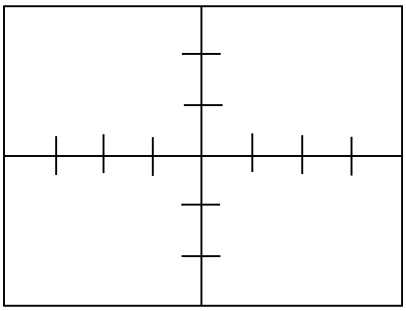
$f(x) = \frac{1}{2}x^2 - 2$



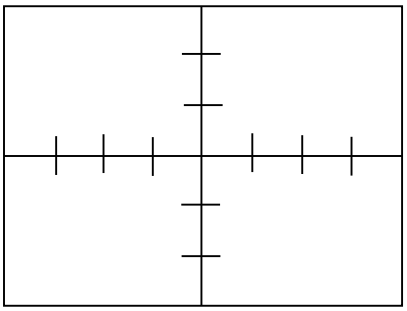
$x = y^2$



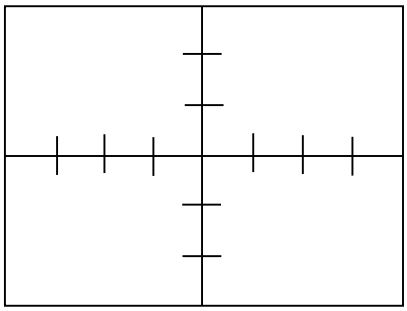
$f(x) = x^3$



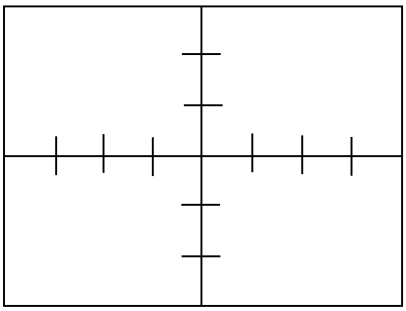
$f(x) = (x-1)^3$



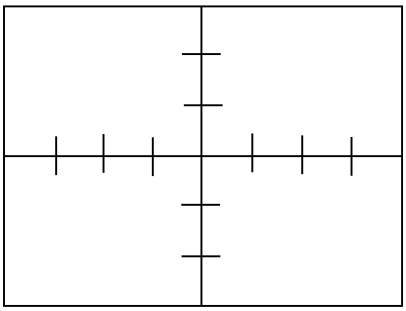
$f(x) = \frac{1}{x}$



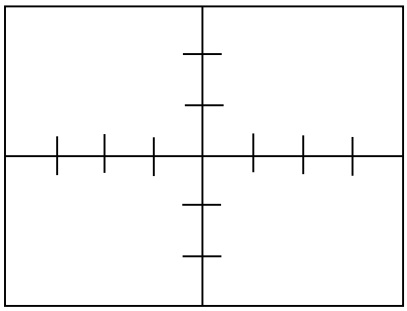
$f(x) = \sin x$



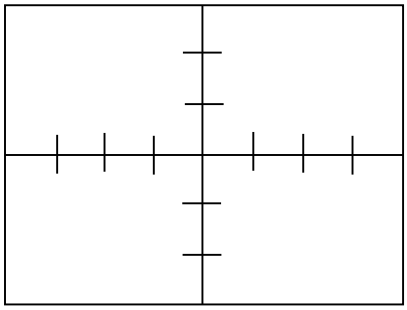
$f(x) = \cos x$



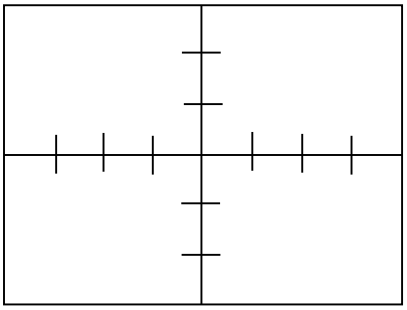
$f(x) = \tan x$



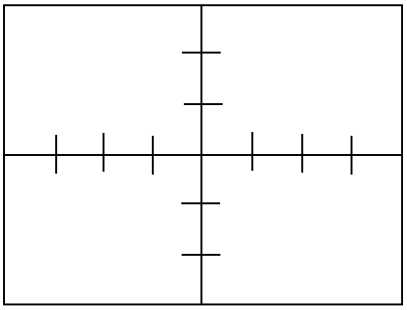
$f(x) = \csc x$



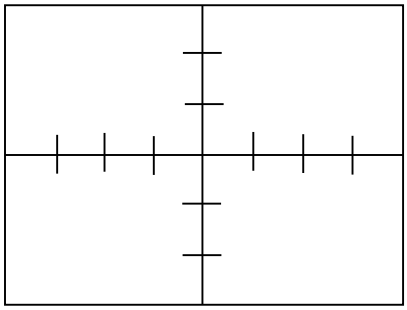
$f(x) = \sec x$



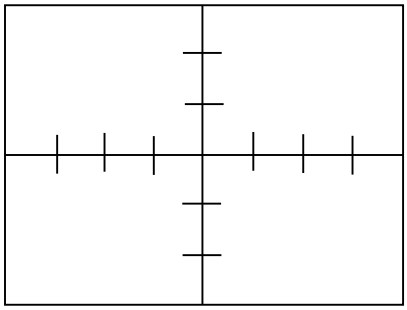
$f(x) = \cot x$



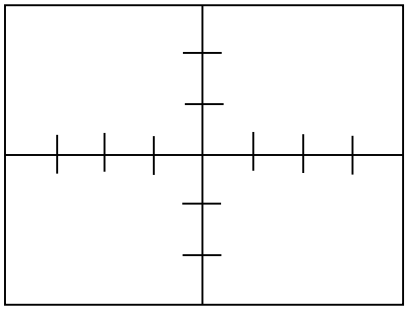
$f(x) = 2^x$



$f(x) = e^x$

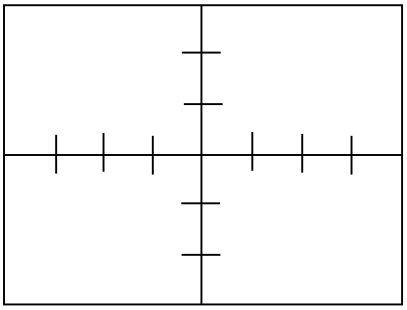


$f(x) = \ln x$

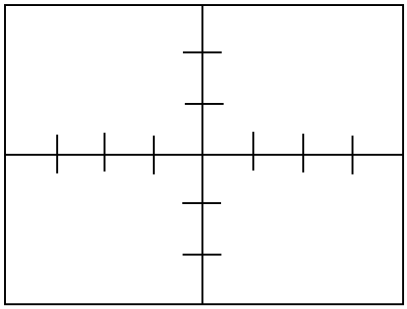


$f(x) = [x]$

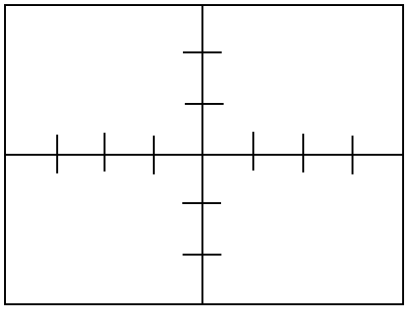
(Greatest integer function)



$f(x) = |x|$



$f(x) = \sqrt{x}$



Part III

FILL IN THE TABLE BELOW. You may use the following abbreviations:
R - the set of real numbers, Z - the set of integers, and N - the set of natural numbers.

FUNCTION	DOMAIN	RANGE	X-INTERCEPTS	Y-INTERCEPTS	ODD OR EVEN	Symmetry
$f(x)=x^2$						
$f(x)=\frac{1}{2}x^2-2$						
$x=y^2$						
$f(x)=x^3$						
$f(x)=(x-1)^3$						
$f(x)=\frac{1}{x}$						
$f(x)=\sin x$						
$f(x)=\cos x$						
$f(x)=\tan x$						
$f(x)=\csc x$						
$f(x)=\sec x$						
$f(x)=\cot x$						
$f(x)=2^x$						
$f(x)=e^x$						
$f(x)=\ln x$						
$f(x)=[x]$						
$f(x)=\lvert x\rvert$						
$f(x)=\sqrt{x}$						